AN EXACT STATISTICAL TEST OF COGNITIVE MAP COMPLEXITY

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Abstract

Despite the growing level of scholarly interest across domains, the implications of cognitive map structural complexity remain poorly understood and we have little insight into the implications and influence of the relative complexity of different map structures. Neither have we had the mechanisms to determine whether any measure of structural complexity picks up anything more than would be detected on a purely random basis. We report a Monte Carlo method of simulation used to empirically estimate parameterised probability outcomes as a means to better understand the behaviour of complexity, and to enhance the development of underpinning theory. We focus on one classic measure of structural complexity, namely link density (or link-to-node ratio), chosen because of its applicability to the wide-range of methods that continue to be developed for the elicitation, construction and analysis of cognitive maps across domains. We illustrate the potential of our work within one context of enquiry, namely the large and growing call centre industry. We demonstrate how the results of our simulation permit the use of exact statistics often prohibited in studies of cognitive mapping – not least because of sample size – which can be applied by hand to an individual map or groups of maps, to gain maximum utility and to support the cumulative process of theory building.

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1.0 Introduction

The notion of a ‘cognitive map’ is essentially a representation of how a person perceives a situation that is their internal representation of ‘reality’. The beliefs that compose these maps provide the individual with a coherent way of organizing and making sense of an otherwise likely confusing array of signals, and a basis for subsequent action (Holsti, 1976). In this paper, we use the term ‘cognitive map’ to represent the essence of a person’s beliefs regarding a particular situation represented in a network based format (Jonassen, Beissner, & Yacci, 1987; Sparrow, 1998). The map depicts constructs and the causal (e.g., Axelrod, 1976; Langfield-Smith & Wirth, 1992) and/or other relationships that a person believes exist between those constructs in a particular domain of interest at a point in time (e.g., Carley, 1997; Nair, 2001).

Known variously as cognitive models, scripts, belief structures, knowledge structures and mental models amongst other terms (Walsh, 1995), cognitive mapping has grown out of a need to capture and articulate these ‘information structures’ because of their – often complex – influence on decision-making, reasoning, predictions about future events, affect and behaviour (see, e.g., Fiske & Taylor, 1991). To this end, mapping has been used extensively and to great advantage in many areas of organization research, for example stress and emotional experience at work (e.g., Health & Safety Executive, 2002), learning (e.g., Carley & Palmquist, 1992), human resource management (e.g., Budhwar & Sparrow, 2002), and technological innovation (e.g., Swan, 1995) and marketing (e.g., Crittenden & Woodside, 2006).

Cognitive maps can be analyzed along two principal dimensions: content, which captures what constructs (variously referred to as concepts, nodes, elements or variables) an individual perceives relevant to a given domain, and structure, which reflects the global organization of those constructs within the map. The content of a map can provide rich insights into the meaning of specific concepts but map comparisons are often difficult because of unique circumstances.

When used for comparative analysis, structural measures can be used to ascertain the different overall map configuration and how simply or complexly a person makes sense of a particular domain (e.g., Carley, 1997; Clarke & Mackaness, 2001; Eden & Ackermann, 1998; Hart, 1976; Jenkins & Johnson, 1997a, 1997b; Nadkarni & Naryanan, 2005; Nair, 2001). At times, a very complex and dense network structure is presented, revealing many links between map constructs and a high level of integration and connectedness, as depicted in Figure 1. In contrast, other cognitive maps with lower link density may present as simple configurations of isolated constructs (see Figure 2).

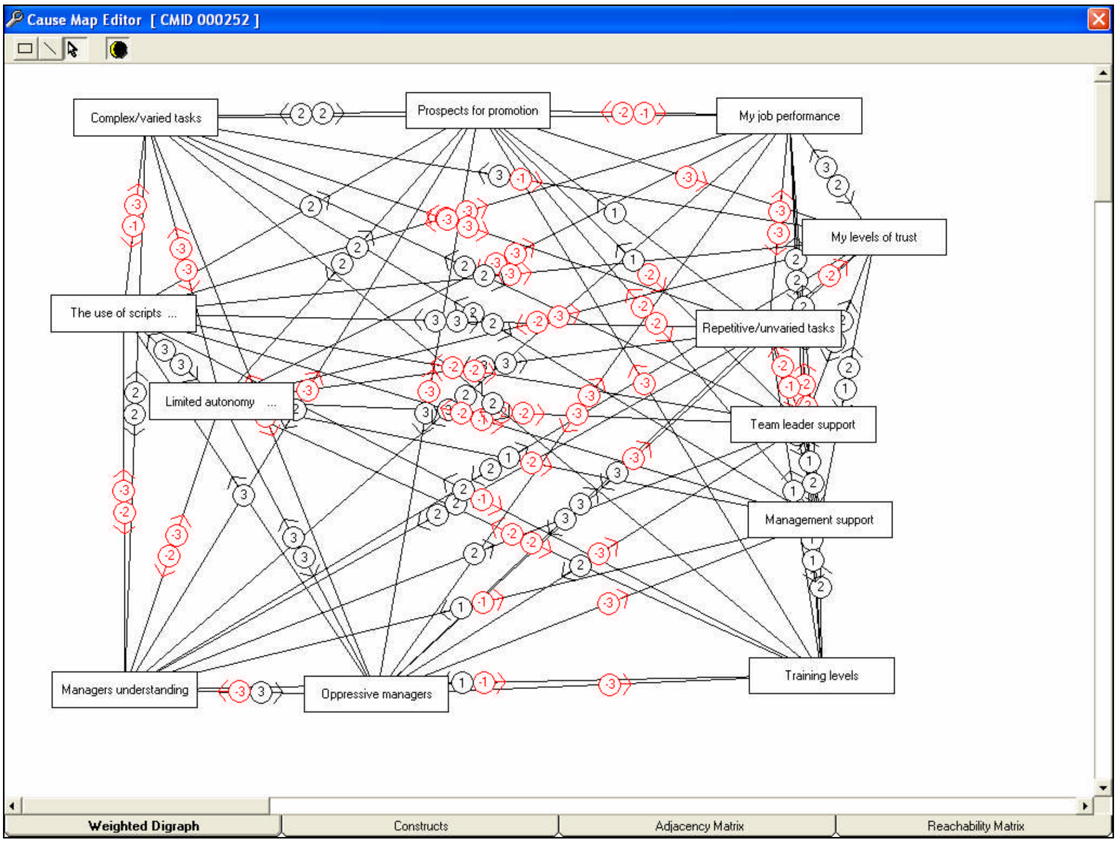

Figure 1: Illustrative visually complex cognitive (cause) map.

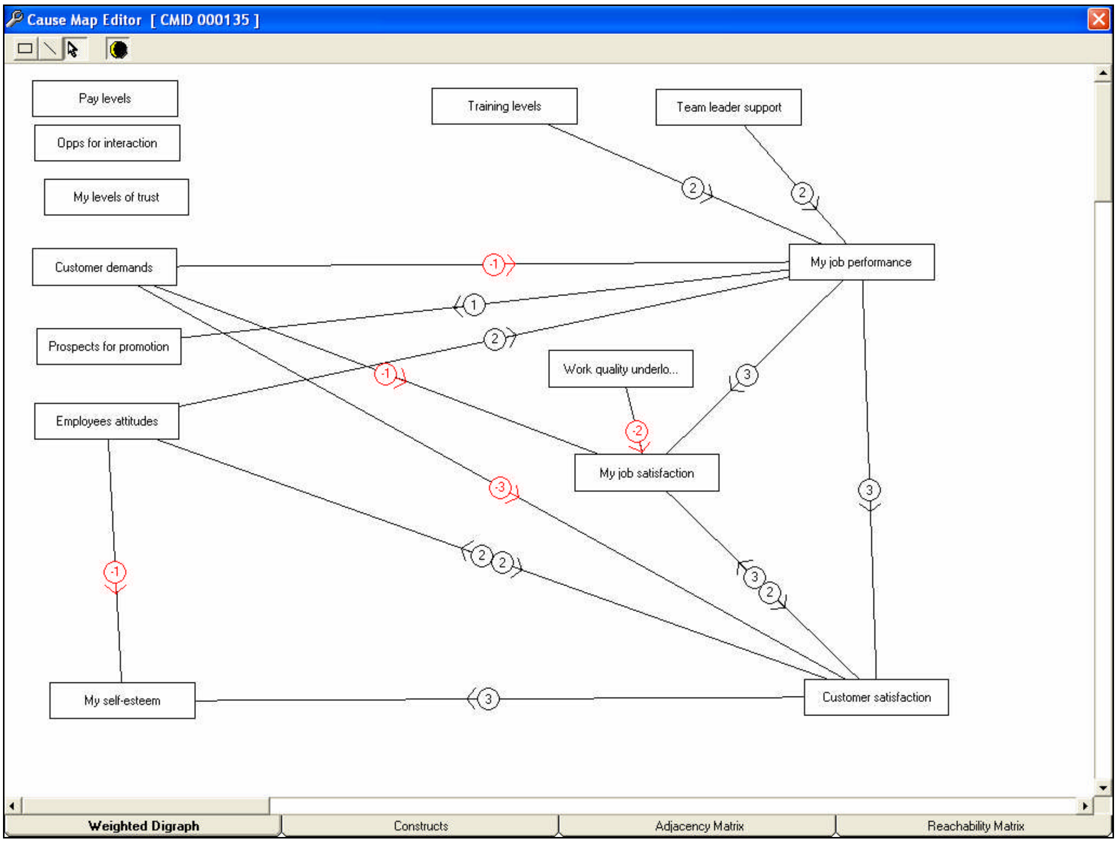

Figure 2: Illustrative visually simple cognitive (cause) map.

The illustrative examples shown in Figures 1 and 2 are cognitive cause maps, shown as weighted digraphs. Within this system of presentation, each perceived causal relationship between a pair of constructs is depicted visually, by means of an arrow-headed pathway, the arrowhead pointing toward the influenced construct (Axelrod, 1976). Depending upon the level of sophistication of the particular variant of mapping employed, links can be differentially weighted. In the examples used in this paper, the magnitude of effect is defined as 1 = increase slight; 2 = increase moderate; 3 = increase strong; -1 = decrease slight; -2 = decrease moderate; -3 = decrease strong.

While visually very different, the implications and influence of the relative complexity of these different structures are not straightforward. A number of areas in the field of managerial and organization cognition (MOC) have now matured to the extent that researchers are seeking to isolate the determinants and predictive value of cognitive maps to test theoretically-driven a priori hypotheses (e.g., Daniels, Johnson, & de Chernatony, 1994, 2002; Markóczy, 1997, 2001; Markóczy & Goldberg, 1995). In other areas, e.g., marketing, and as expressed by Christensen and Olson (2002, p.499) “The process of mapping consumers’ mental models is still in its infancy. Thus there are numerous opportunities for future research to improve both the methods and interpretation of cognitive structure analysis”.

In his detailed summary of the managerial and organizational cognition literature, Walsh (1995) concluded that research into map structure fell way behind that of content, with a recommendation that more work should be carried out to substantiate the tentative findings to-date. However, and despite many additional calls for research that will permit a better understanding of the determinants of cognitive map complexity, its implication for behavior and performance and the importance of context in this regard (e.g., Calori, Johnson, & Sarnin, 1994; Clarke & Mackaness, 2001; Nadkarni & Narayanan, 2005; Nair, 2001), empirical studies fall significantly below the level of interest.

Axelrod (1976) developed a cognitive mapping method designed to capture the qualitative causal relationships thought to represent the structure of political decision problems. Some simple inference mechanisms – based on path analysis from cause to effect – were proposed to draw conclusions about the consequences of a cognitive map. However, cognitive mapping practice in political science and beyond has continued to be primarily for descriptive purposes and, in order to judge whether the conclusions being drawn regarding the implications of cognitive map structural complexity are valid and under what conditions, we need to advance our understanding of the implications of cognitive mapping from qualitative description to a hypothesis-testing mode.

Undoubtedly, a key reason for this lack of research concerns the labour intensive nature of most forms of cognitive mapping techniques (see, e.g., Huff, 1990; Huff & Fletcher, 1990; Nair, 2001), which is prohibitive of the collection of the type of large-scale samples typically required to enable statistical inference to be drawn. Thus, we need to develop methods by which to permit inferential statistics to be performed not only on large-scale samples but also on a single map or on small groups of maps, in order to support the cumulative process of theory building.

In this paper, we report the results of a Monte Carlo method of computer simulation that was carried out in order to estimate the probability distribution of link density, i.e. the proportion of links to the number of constructs within a map, or link-to-node-ratio (see, e.g., Calori et al., 1994; Carley, 1997, Carley & Palmquist, 1992; Eden, Ackermann, & Cropper, 1992; Laukkanen, 1994). This classic measure of complexity was chosen because of its wide applicability to the many mapping methods that continue to be developed for the elicitation, construction and analysis of cognitive maps. Our simulation method was also chosen to provide maximum utility, as the results can be applied by hand, thus making our significance tests accessible to organizational researchers across domains.

Following the next section of this paper, in which we summarize the theoretical, philosophical and practical context, and importance of this work, we explain how link density is calculated together with a simple interpretation of its meaning and behaviour. In the subsequent section, we provide details of our simulation and the formulas derived. Knowing these underlying distributions permits the use of statistics often prohibited in studies of cognitive mapping – not least because of sample size. For example, basic assumptions for simple t-tests include the fact that the scores obtained for each group should be normally distributed and when this assumption is violated there can be very serious consequences, the most obvious being that the probabilities associated with a normal distribution are not valid. While – and at the risk of oversimplification - for sample sizes of 30+ this is unlikely to cause very serious problems, many studies of cognitive mapping fall well below this number. While non-parametric tests do not have such stringent requirements and do not make assumptions regarding the underlying distribution, they tend to be less sensitive than their parametric counterparts and therefore may fail to detect differences between groups that actually do exist. However, knowing these underlying distributions permits the use of more accurate statistical comparisons using the z-test and the ability to gain additional depth of insight. Drawing upon data from one research context, we illustrate the application of these formulas and their potential to contribute to the development of theory. We conclude by outlining future extensions of this line of work but first, in the following section, we set the wider context.

2.0 Theoretical, Philosophical and Practical Context

The earliest work on cognitive maps is generally credited to Tolman (1948). Since that time, the term ‘cognitive mapping’ has captured the interest of researchers from a range of disciplines – not least psychology, education, and many areas of management – and it is unsurprising that Ackermann (2008, p.42) recently concluded “As a consequence there has emerged many different forms of cognitive mapping, each with their own theoretical, philosophical and practical bases”. For some, cognitive maps are viewed as being capable of representing an individual’s literal beliefs concerning a particular domain at a given point in time (e.g., Langfield-Smith & Wirth, 1992), with the potential to have the same essential characteristics as thought itself (e.g., Huff, 1990). While others disagree, a number of organizational researchers maintain that this is largely irrelevant and that mapping need not necessarily be linked with the cognitive map construct – as developed in the field of psychology – to be a meaningful way of representing elements of thoughts (rather than thinking) (e.g., Eden, 1992; Hodgkinson & Clarkson, 2005; Laukkanen, 1998; McDonald, Daniels, & Harris, 2004). As pointed out by Nelson, Nelson and Armstrong (2000: 1), in the context of the capture of information systems expertise, it is not possible to literally “open the expert’s head” and extract domain knowledge as represented directly in the human brain. It follows that methods are required that can represent knowledge in ways that capture the essence of peoples’ thoughts and belief systems. To this end, there has been a proliferation in the development of cognitive mapping techniques, which we briefly review in following section to provide sufficient context for, and to demonstrate the wide applicability of, our contribution.

2.1 Methods by which to elicit and construct cognitive maps

Beyond fundamental epistemological considerations, decisions regarding the most appropriate method by which to elicit and construct cognitive maps depend upon whether the model of cognition is seen to be relatively simple, where, for example, simple counting and weighting of words in a text (Berelson, 1952) would be acceptable, or rather more complex. (See Huff, 1990 for a description and succinct summary of the continuum of choices). The ability to represent complex and rich information without imposing a linear structure is a key advantage of many mapping methods and, without doubt, most common forms reveal dynamics, and understanding of perceived influence – often causality – that a person uses to understand organizational (as in our illustrative examples) or other situations.

Indirect elicitation techniques entail processes whereby maps are constructed from secondary data sources, typically written documentation, including interview transcripts derived initially for some other purpose then subsequently analysed using cognitive mapping procedures (e.g., Barr & Huff, 1997; Barr, Stimpert, & Huff, 1992), without the active involvement of the research participant. In contrast, direct elicitation methods require the active involvement of participants in the map construction process, from the outset.

Direct elicitation procedures can be sub-divided in terms of the extent to which the elicitation process requires participants to identify the constructs to be mapped, using their personal everyday natural language, or whether the subject matter is supplied by the researcher, on the basis of theory and research or an a priori conceptual analysis of the domain to be mapped. The primary concern of researchers adopting an idiographic approach – for example the use of hand-drawn cognitive maps - is to capture participants' thoughts in their natural language form, thereby minimising the dangers of researcher bias in the knowledge elicitation process and ensuring that the richness and detail of individual cognition is preserved (see e.g., Eden & Ackermann, 1998). In contrast, nomothetic approaches entail the use of standardized lists of variables supplied by the researcher in the form of, for example, highly structured questionnaires (Roberts, 1976). A primary strength of this approach is that the comparative analysis of the resulting maps is considerably less problematic but a major drawback is that the basic elicitation task might prove meaningless for participants.

'Hybrid' approaches seek to combine the strengths of idiographic and nomothetic approaches while dispensing with their associated weaknesses. One such approach, devised by Markóczy and Goldberg (1995), entails the development of a large and inclusive pool of constructs representative of the knowledge domain under investigation. Using this procedure, participants are required to select a sub-sample of personally salient constructs to be incorporated within their map (typically 10-13 in number). The primary advantage of this technique is that it ensures the knowledge elicitation task is personally meaningful to the individual participant, but also ensures that the resulting maps are directly comparable on a structural (and content) basis. It has been argued that since such methods afford the use of larger sample sizes they will increase the replicability of findings (e.g., Hodgkinson, 2001) and recent advances in computer software support this intention (e.g., Clarkson & Hodgkinson, 2005). Others argue that an increase in quantification and standardisation in terms of research instrument will not necessarily equate to a more exact replication of cognition (e.g., Daniels & Johnson, 2002) and can limit research to the preconceived ideas of the researcher and hence introduce a different kind of bias whilst sacrificing richness in the data. However, as noted by many scholars, the nature of the solution is often appropriately rooted within the research questions and approach adopted within a given study (e.g., Daniels & Johnson, 2002; Huff, 1990; Jenkins, 1998). While a detailed consideration of these is not possible within the confines of the present paper, this issue was fundamental in our considerations and our choice of structural measure – link density – was chosen because it is not affected by the restriction of constructs required in many methods of cognitive mapping, and thus has strong potential to be utilised to move towards resolving the long-standing (often circular) debates related to cognitive complexity – some of which we briefly consider in the following section.

2.2 Implications of relative map complexity and simplicity

Schroder, Driver and Streufter (1967) suggested that the level of cognitive integration is an indicator of an individual's information processing capability, which is crucial to generating diverse perspectives and explanations of a situation or problem. Many scholars conclude that the complexity of map structure may provide a means to examine the variety in and development of more complicated thinking (e.g., Bartunek, Gordon, & Weathersby, 1983; Eden & Ackermann, 1998; Eden et al., 1992; Hart, 1976). Following from this, it has largely been suggested that a simply structured map will have negative consequences, equating this simplicity with narrowness of vision and an impoverished view of the work (e.g., Bartunek et al., 1983; Miller, 1993; Weick, 1969). In contrast, some have gone on to question whether very elaborate or highly dense cognitive maps might invite a form of 'paralysis by analysis' (Hodgkinson & Maule, 2002) and, in fact, be dysfunctional (Nair, 2001).

Research has produced equivocal results (e.g., Amernic & Beechy, 1984; Calori et al., 1994; Carley, 1997; Carley & Palmquist, 1992; Hall, 1976, 1984; Nair, 2001; Varma & Krishnan, 1986). Nair (2001) revealed that, while link density was significantly related to the effectiveness with which 45 managers individually solved a simulated complex problem and effectiveness increased with increase in the elaborateness of their cognitive map, this increasing trend was only evident up to a point, after which effectiveness declined with further increase in density. In a similar vein, Calori et al. (1994) found that the top managers of firms with an international, rather than national, geographic scope had more complex maps of the structure of their environment and concluded that complex maps do not necessarily lead to superior performance, rather maps should display "requisite cognitive complexity" (p.439). Unfortunately, only relatively modest amounts of variance were explained with moderately low levels of statistical significance in this 26 participant exploratory study, but the results were encouraging and suggested the need for larger sample sizes in future investigations.

Walsh (1995: 300) questioned whether additional accountability might – in itself – prompt people to think in a complex manner. Clarke & Mackaness' (2001) exploratory study revealed the cognitive maps of senior managers to be no more complex and coherent than their less senior counterparts. The authors go on to tentatively suggest (bearing in mind their study involved just three participants) that senior executives attempt to simplify – vs. making more complex – a situation (in this instance their approach to decision-making), but that this would require testing through "extensive research".

Nadkarni and Naryanan's (2005) recent investigation of 204 students found a strong relationship between the level of map complexity and cognitive ability, and with academic test grades. They also revealed the influence of context-related factors on levels of complexity and, overall, their results suggested that complexity is not a general cognitive attribute, rather it is indicative of domain knowledge. The results of Nadkarni and Narayanan's study raised further questions and suggestions that future research should investigate the role of complexity to facets of performance such as employee reliability, work quality, and interpersonal orientation.

In all, there is a clear mismatch between scholarly interest and empirical investigations, rendering underpinning theoretical frameworks weak, contradictory and unsupported. Undoubtedly one explanation relates to the labour intensive nature of most forms of cognitive mapping techniques (see, e.g., Huff, 1990) and, while subjective judgments can be made regarding the significance of measure values, the statistical significance of these structural measures cannot be determined, as their underlying distributions are unknown. Theoretically derived statistical models reflecting some aspects of directed graphs have been developed (see, Erdös & Rényi, 1961; Holland & Leinhardt, 1981) but, to our knowledge, no models have been devised which take into account all the features defined in this paper. However, simulation provides a feasible way to empirically estimate parameterized probability outcomes. In this paper, we advance understanding of the implications of cognitive map structure by estimating the probability distribution for the structural measure of cognitive map complexity – link density – by simulation using a Monte Carlo method but first, n the next section, we provide details of how link density of cognitive maps is calculated together with a simple interpretation of its meaning and behaviour.

3.0 Link Density

In demonstrating how link density is calculated, we use nine definitions:

Definition 1: c represents a construct within a map.

Definition 2: C is the number of constructs in a construct pool (where participants are presented with a predetermined list or pool of constructs). C can be $>= 0$ and $< +\infty$.

Definition 3: c_i represents construct i in a map. The indexer i can take on values of 1 to C.

Definition 4: $s(c)$ represents if a construct is selected. $s(c) = 1$, when a construct is selected. $s(c) = 0$, when a construct is not selected.

Definition 5: S is the number of selected constructs in a map. S must be $<= C$. (In methods where participants do not choose from a list or pool S will always $= C$).

Definition 6: $s(c_i)$ represents if construct i is selected. The indexer i can take on values of 1 to S.

Definition 7: l represents a link between two constructs. A construct cannot be directly linked to itself. A link can only exist between two selected constructs, where a selected construct is defined as in Definition 4. A link has a direction, it is said to link a source construct to a destination construct. A single construct may act as the source for many links. A single construct may act as the source and/or destination for many links.

Definition 8: L is the number of links in a map. L can be $>= 0$. If there are S constructs then each individual construct can have at most $S - 1$ links. For S constructs the maximum number of possible links within a map must therefore be $S * (S - 1)$. Therefore $L <= S * (S - 1)$.

Definition 9: l_i represents link i in a map. The indexer i can take on values of 1 to L.

To illustrate these definitions Figure 3 presents a simple cognitive map. There are 5 constructs within the construct pool; $C = 5$. There are 4 selected constructs within the map; $S = 4$. There are 5 links within the map; $L = 5$.

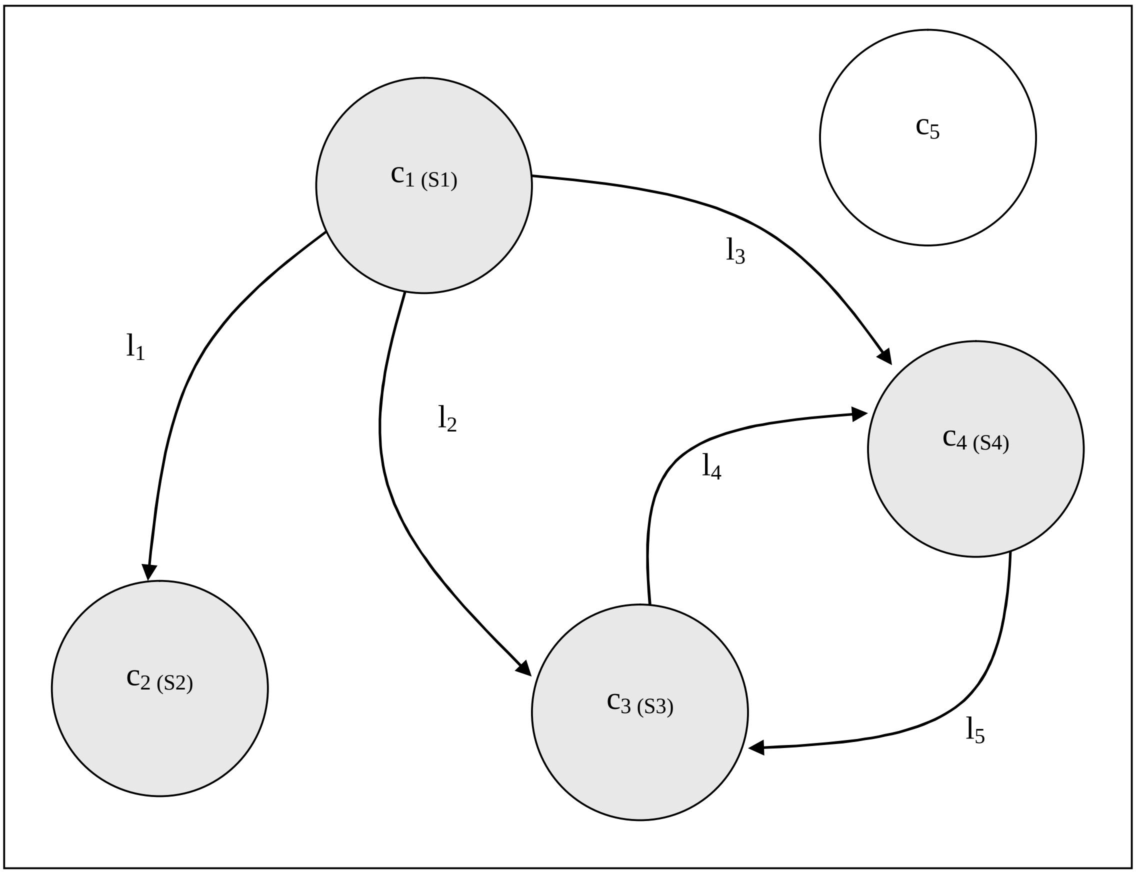

Figure 3: Simple illustrative cognitive map, shown in the context of the overall construct pool (C 1 - 5). Selected constructs incorporated in the map (S 1 - 4) are shaded.

Definition of Link Density: $LD = \frac{L}{S}$

When `S = 0`, `LD` is undefined.
For `S` constructs there is a theoretical maximum of `S * (S – 1)` links. Therefore:

$$\text{Minimum value}\,(\text{LD}_{\text{min}}) = \frac{0}{S} = 0$$

$$\text{Maximum value}\,(\text{LD}_{\text{max}}) = \frac{S*(S-1)}{S} = S-1$$

A simple interpretation of the meaning and behaviour of link density is that if it is close to 0 then the cognitive complexity, connectedness and integration is low for the cognitive map. In contrast, if link density is close to LD_{max} then the cognitive complexity, connectedness and integration is high for the cognitive map. In our next section, we present the results of a Monte Carlo method of simulation, which was conducted to provide probability distributions for link density within any given map, to support a better understanding beyond these simple interpretations.

4.0 Simulation

4.1 Method

The simulation model is based on the Monte Carlo method (Metropolis & Ulam, 1949). As we have noted, this method was chosen largely on the grounds of practicality, i.e., any results of this form of simulation could be applied by hand providing maximum utility in an area that is attracting increasing scholarly interest across domains. This would be impractical using a Bootstrapping method, and would potentially prohibit the use of significance tests by many organizational researchers.

The model consists of a series of actions (a – f), which are executed in order to create a simulated cognitive map.

(a) The number of constructs (`C`) in a simulated cognitive map is defined.

(b) All constructs are selected. Therefore the number of selected constructs (`S`) in the simulated map is `S = C`.

(c) For each possible ordered pair of constructs, excluding pairing a construct with itself, the presence or absence of a link is decided by randomly selecting an integer from the uniform distribution of values `0` and `1`. If value `0` is selected, a link is not added between the ordered pair of constructs. If value `1` is selected, a link is added between the ordered pair of constructs. An ordered pair of constructs consists of a source and a destination construct. A link will be from a source to a destination construct. A separate link can also exist between a destination and source construct.

(d) For each link the sign of the weight is decided by randomly selecting an integer from the uniform distribution of values `0` and `1`. If value `0` is selected, the link weight is negative. If value `1` is selected, the link weight is positive.

(e) For each link the absolute weight is decided by randomly selecting a real value from the uniform distribution of real values between `0` and `1`, where value `0` is excluded and value `1` is included.

(f) For each link the weight is calculated by multiplying the randomly generated absolute weight by the randomly generated weight sign.

4.2 Simulation experiments

The model was implemented as a bespoke application developed by the authors. Twenty separate simulation experiments were carried out. For each experiment, 100,000 simulated cognitive maps were generated. For each simulated map within an experiment, link density was calculated and a frequency distribution of the simulated values was counted. Taking the minimum and maximum value of the simulated link density, and dividing this range into 45 equal intervals defined the frequency distribution. All the simulated measure values (i.e. 100,000 for each experiment) were then classified into these intervals.

4.3 Validation of random number generator

Our bespoke simulation application employed the random number generator *System.Random* class from the Microsoft .NET Framework 1.1, which, following the recommendations of Law and Kelton (2000), was validated by four tests of randomness (Chi-square; Serial; Runs-up; Correlation). This presented sufficient empirical evidence to accept the validity of the *System.Random* random number generator from the Microsoft .NET Framework 1.1

4.4 Simulation results and analysis

The simulated frequency distribution for each experiment was examined to determine its fit to the normal distribution. For large n (100,000 in this study) it is commonly acknowledged that any goodness of fit statistic such as Lilliefors will give rejection of the null hypothesis of normality and that skewness and kurtosis are too sensitive to provide meaningful significance testing. Therefore, for each simulation experiment the frequency distribution was visually inspected and, in each instance, the histograms, boxplots and Q-Q were judged to display normality characteristics. Having established normality for all simulated distributions, for each simulation experiment the mean (Table 1) and standard deviation (Table 2) of the frequency distribution for link density was calculated.

No of mapped constructs	Mean of frequency distribution	No of mapped constructs	Mean of frequency distribution
5	2.0054	55	27.0037
10	4.5008	60	29.4990
15	6.9998	65	32.0001
20	9.4987	70	34.4974
25	11.9995	75	36.9950
30	14.5000	80	39.4996
35	16.9978	85	41.9991
40	19.4999	90	44.4991
45	21.9994	95	46.9992
50	24.5012	100	49.5027

Table 1: The mean frequency distribution for link density.

No of mapped constructs	SD of frequency distribution	No of mapped constructs	SD of frequency distribution
5	0.4210	55	0.4978
10	0.4947	60	0.4986
15	0.4750	65	0.4973
20	0.4855	70	0.4996
25	0.4948	75	0.4996
30	0.4982	80	0.5001
35	0.4958	85	0.5002
40	0.4961	90	0.4971
45	0.4954	95	0.4995
50	0.4966	100	0.4966

Table 2: The standard deviation of the frequency distribution for link density.

A linear and non-linear regression was then carried out for the means and standard deviations to determine formulae that can be used to compute the expected mean and standard deviation of a population of random cognitive maps for a given value of S.

The independent variable (X) used was the number of constructs. The dependent variable (Y) used was either the mean or standard deviation of the frequency distribution of link density. In the absence of theory to inform choice of the best candidate regression, we employed a strategy of pragmatism and parsimony. We identified a standard set of regression models – ordered from the most simple to more complex – namely: linear ($Y = b_0 + b_1X$); inverse ($Y = b_0 + b_1/X$); quadratic ($Y = b_0 + b_1X + b_2X^{2)}$; cubic ($Y = b_0 + b_1X + b_2X^2 + b_3X^3$); and power ($Y = b_0\ X^{b1).}$ As can be seen in Table 3 and Figure 4 a good statistical fit was identified (using simple linear and inverse models).

Mean	Standard Deviation
$R^2 = 1.000$ 18 df $F = 8.5E^8$ $p = 0.000$ Linear model $b_0 = -0.4991$, $b_1 = 0.5000$	$R^2 = 0.846$ 18 df $F = 99.25$ $p = 0.000$ Inverse model $b_0 = 0.5052$, $b_1 = -0.3651$

Table 3: Regression curve estimate parameters.

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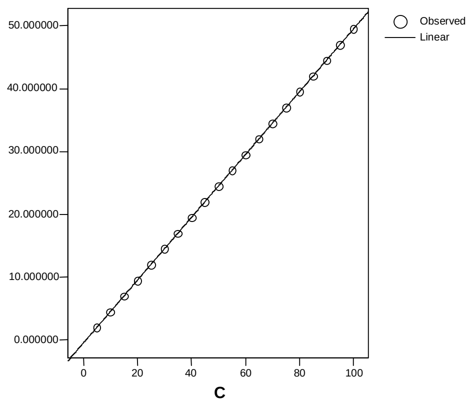
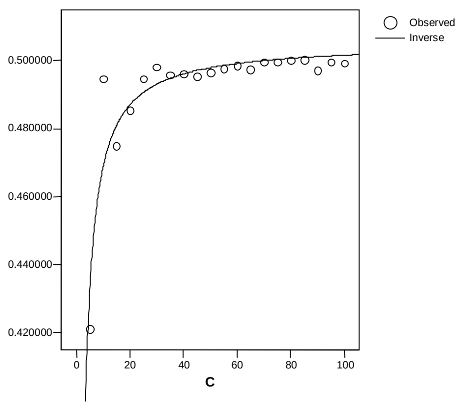
<tr><td></td><td>
</td><td>
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Figure 4: Regression curve estimate plots.

The formulas that allow us to calculate the expected mean and standard deviation values for a population of random cognitive maps for a given value of S are:

Formula X
Mean = -0.4991 + 0.5000 * S

Formula Y
StdDev = 0.5052 - 0.3651 * S

In our next section, we illustrate the application of these formulas in study context.

5.0 Illustrations of the Method

5.1 Context and mapping procedure

We gathered our data in the context of the large, and growing, call centre sector, which is typically characterized by the management of front line operatives at one central point and represents a mass production approach to customer service (Batt, 1999). The front line employee spends a significant proportion of working time responding to or initiating calls on the telephone while simultaneously using display screen equipment to recall or input organizational and/or client information. The front line agent is typically suggested to be working in a routine, highly controlled, low discretion environment and concerns have been expressed not only regarding low productivity but also the well being of front line employees (e.g., Deery, Iverson, & Walsh, 2002; Taylor, Mulvey, Hyman, & Bain, 2002). The team leader role is viewed by some as the most important and most pressurised in the call centre environment. Not only does the role entail supervision and the assumption of responsibility and accountability for the actions of their team members but, at times, the requirement to stand in for, and carry out the same duties as, the front line agents in their team. Our data were gathered from 200 call centre employees - 178 front line call centre agents, and 22 team leaders, and our aim was to ascertain what each individual felt were the most salient features (constructs) in their employing call centre and to identify any perceived relationships (links) between these constructs.

We employed a variant of the hybrid approach to cognitive mapping devised by Markóczy and Goldberg (1995). First, we developed a large and inclusive pool of constructs from a detailed review of diverse range of adjacent literatures from the field of industrial, work and organizational psychology (e.g., Clegg & Wall, 1990), together with specialist call centre research (e.g., Holman, 2002), in order to identify theoretically informed constructs of likely salience to our call centre front line participants. Our data were gathered through a series of individual interviews using CognizerTM, a specialist computer software package devised to facilitate the elicitation, analysis and comparison of cause maps (Clarkson & Hodgkinson, 2005). Within this system, participants choose from an onscreen menu a subset of personally salient constructs for mapping, presented in random order in order to minimize potential order effects. From a pool of 55 theoretically derived constructs (C=55), the participants were requested to chose a set of 10-13 constructs (S) representing the features they felt were of most salience in respect of their employing call centre. They then systematically considered if and in what way the chosen constructs were (perceived to be) related (in this instance causally) to one another. As it was recognized that differences in length of interview could bias complexity – the longer the interview the more complex the map – with team leaders potentially having more control in permitting a lengthier interview, interview length was standardized for all participants.

5.2 Illustration of technical contribution

As noted earlier, Walsh (1995: 300) questioned whether accountability might prompt people to think in a complex manner, while Calori et al. (1994: 439) concluded that cognitive maps should display “requisite cognitive complexity”. As the call centre team leader’s role will be relatively more complex and involve a greater level of accountability than that of the typical front line agent, this would suggest that job role will make a statistically significant difference to structural complexity - team leaders will have more complex cognitive maps than their front line team members. The work of other scholars would, however, lead us to contrary conclusions. Schroder et al. (1967), for example, suggested that the level of cognitive integration is an indicator of an individual’s information processing capability, rather than being representative of that person’s domain or role.

The descriptive statistics for link density for our two groups of employees are shown below:

Role	N	Mean	SD
Front line agent	178	3.6571	1.7602
Team leader	22	4.9623	2.2009

With no explicit knowledge of the underlying population distribution, the normal statistical approach to test our hypothesis is to use an independent sample t-test (as long as the underlying assumptions are satisfied - noted above). For our example:

	T	Df	p (2-tailed)
Where equal variance is assumed (Levene’s test for equality of variances significance value = .398)	-3.1872	198	0.0017

The t-test therefore revealed a statically significant difference for the average link density value between the groups. However, as we now know the underlying population distributions we can use an exact statistical test, which is the two-sample z-test. Before we can use this test, we must transform our variable – link density – as the number of mapped constructs determines the underlying population distribution, and in our example data (and as in most other empirical maps) the number of mapped constructs varies from participant to participant. The most obvious transformation to use is to convert link density to z values by submitting the population mean (calculated from our formula X) and dividing by the population standard deviation (calculated from our formula Y). The new transformed variable now has the standard normal distribution (population mean = 0, standard deviation = 1). We illustrate below using one cognitive map where:

S (number of selected constructs)	= 13
LD (Link density for selected constructs)	= 4.08
μ	= -0.4991 + 0.5 * 13
	= -0.4991 + 6.5

$= 6.0009$

$\sigma \quad = 0.5052 - 0.3651 * 13$
$= 0.5052 - 4.7463$
$= -4.2411$

$$Z = \frac{LD - \mu}{\sigma} = \frac{4.08 - 6.0009}{-4.2411} = \frac{-1.9209}{-4.2411} = 0.45$$

Descriptive statistics for the transformed variable for the two groups of employees are shown below:

Role	N	Mean	SD
Front line agent	178	0.4495	0.4588
Team leader	22	0.1309	0.5621

The two-sample z-test under the null hypothesis reveals $z = 1.4097$; $p = 0.1586$ (2-tailed). Therefore – contrary to the earlier t-test – our z-test reveals no statistically significant difference in link density between the two groups of call centre employees.

5.3 Illustration of potential to gain further depth of understanding

Call centre employees have been typically educated to GCSE O' (basic academic) level standard but industry literature suggests large and increasing numbers are qualified to degree standard and there are specific instances of professionally skilled workers, for example nurses and HR professionals, working on the front line. It has been suggested that the complexity of map structure may provide a means to examine the development of more complicated thinking (e.g., Bartunek et al., 1983; Eden & Ackermann, 1998; Eden et al., 1992; Hart, 1976). This would suggest that, despite the simple, routine and controlled call sector context, the more educated the worker the greater the overall complexity of their cognitive map. Alternatively, as we have spoken of earlier, the results of other researchers suggest that complexity is indicative of domain knowledge, leading to rather a different proposition, i.e. that individuals are likely to accommodate this simplicity as revealed in their cognitive maps, with no distinguishable differences associated with level of education.

Descriptive statistics for our 200 participants (Chi-square test for independence, revealed no difference in the proportion of team leaders and front line agents holding the various levels of qualification) revealed that mean link density scores increased with level of education:

Level of education (highest qualification = proxy variable)	N	Mean	SD
Group 1: No qualifications	11	2.81	1.57
Group 2: GCSE (basic academic) or equivalent	96	3.53	1.50
Group 3: A (advanced academic) or equivalent	62	3.89	1.81
Group 4: Degree or equivalent	31	4.82	2.59

A one-way between-groups analysis of variance (ANOVA) was conducted to explore the impact of level of education (as measured by the proxy variable 'highest level of qualification') on link density. The ANOVA revealed a statistically significant difference in link density between the four levels; $F_{3, 196} = 5.131$, $p = 0.002$. Thus, our findings suggest that that the more educated the worker the greater the overall complexity of their cognitive map. However, utilizing our z scores to compare with a random set of cognitive maps, provided additional information, whereby – and contrary to either of our stated expectations - the *simplest* maps were associated with the highest levels of education:

Level of education (highest qualification = proxy variable)	N	Mean	SD
Group 1: No qualifications	11	0.69	0.38
Group 2: GCSE (basic academic) or equivalent	96	0.48	0.36
Group 3: A (advanced academic) or equivalent	62	0.39	0.49
Group 4: Degree or equivalent	31	0.14	0.63

This is depicted in Figure 5.

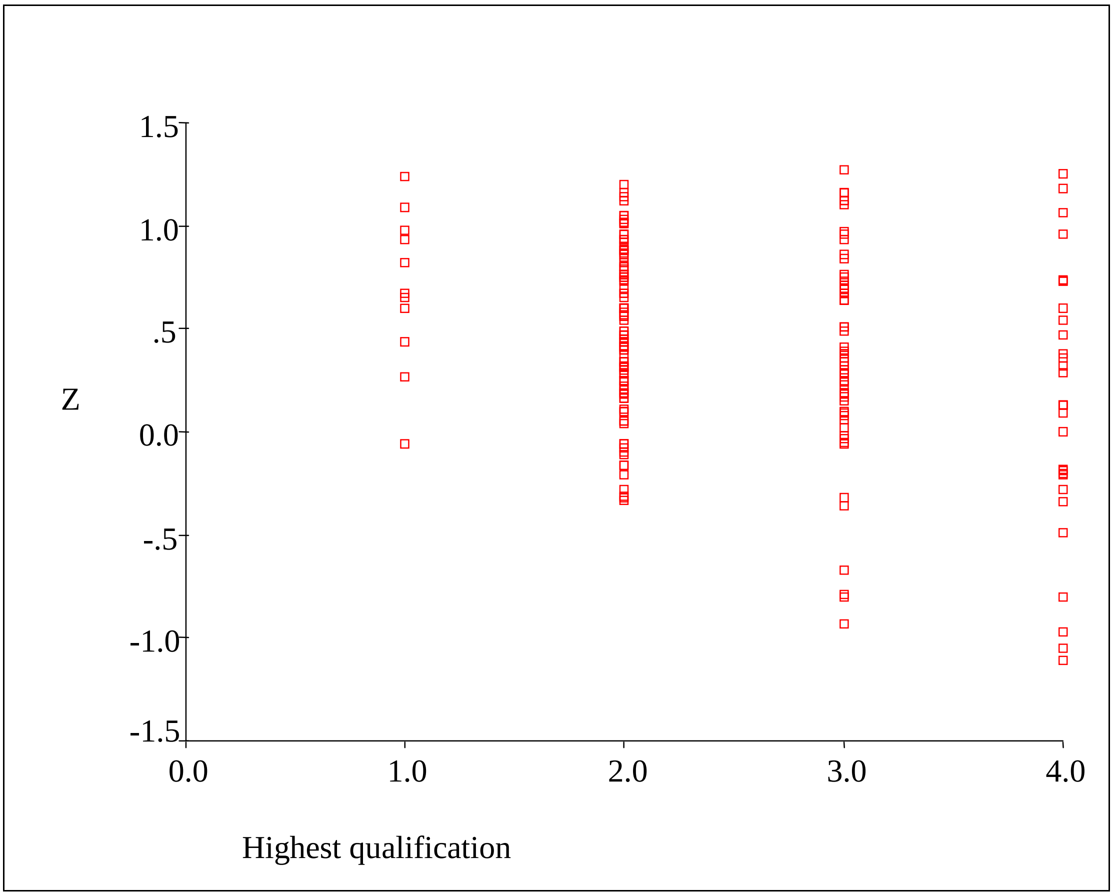

Figure 5: Association of link density z scores with level of education. Z = link density z scores. 1 = no qualifications; 2 = GCSE (basic academic); 3 = A (advanced academic); 4 = Degree.

6.0 Discussion

We conducted a Monte Carlo simulation to empirically derive probability distributions for one classic measure of structural complexity, namely link density. After establishing normality for all simulated distributions, for each simulation experiment the mean and standard deviation of the frequency distribution was calculated and then fitting regression models we derived formulae that can be used to compute the expected mean and standard deviation values for the measure for a population of random cognitive maps. As we now know the underlying population distributions we can use an exact statistical test, thus supporting researchers to determine whether the conclusions being drawn regarding the implications of cognitive map structural complexity are valid and under what conditions.

Our two illustrative examples utilized the maps of 200 call centre employees, but our worked transformation – where we converted link density to a z value (section 5.2) - demonstrated the applicability of our formulas to a single map or to small numbers of maps. As seen in our first illustrative example, contrary to an earlier t-test, the two-sample z-test revealed no statistically significant difference in link density between two groups of call centre employees, front line agents and team leaders. Thus – and at the risk of gross oversimplification – our first test supported the propositions that accountability in itself might prompt people (e.g., team leaders) to think in a complex manner, and/or that cognitive maps become more complex on a needs must basis, e.g., relative to job role. This support was subsequently overturned, as the results using our exact statistical test gave support to the suggestion that level of cognitive integration is an indicator of an individual's information processing capability, rather than being representative of, for example, an individual's role within the organization.

The results of our second illustrative study – concerning the influence of level of education – provide very insightful information. We had actually examined our data in line with two lines of thought based on current understanding. The first suggests that, as the complexity of map structure may provide a means to examine the development of more complicated thinking, increasing levels of education will be associated with corresponding increases in link density, i.e. more educated employees will have more complex maps. A second line of reasoning, that complexity is indicative of domain knowledge, indicated that we would not find this pattern, essentially the maps of our more educated employees would be no more complex than their less educated colleagues. Our results, in fact, support a more complex suggestion - that while the level of cognitive map complexity may be an indicator of the development of an individual's information processing capability, this does not necessarily mean that this will manifest in increasing complexity but, in fact, in simplicity. It may be that our more educated participants were able to simply 'cut to the chase' – providing some support to the tentative conclusions of Clarke and Mackaness (2001) – or, given the relative simplicity of the call centre context, display 'requisite cognitive complexity' in support of the line of reasoning of Calori et al. (1994).

There may, of course, be alternative explanations for our results. For example, it could have been that our more educated participants were more disengaged with the mapping process – though there was nothing indicative of this at the time of data collection. In addition, many other situational and/or individual difference factors – beyond the role of level of education - could have exerted significant influence on the complexity levels of these two groups of employee

However, we have not argued in support of any alternative explanation (neither was it our intention). We have simply demonstrated the usefulness of our contribution to move towards a better understanding of the behaviour of structural complexity and – ultimately - what can we predict from it. More than thirty years ago early empirical research supported the predictive values of cognitive maps more generally (see Bonham & Shapiro, 1976) yet, despite the growing interest in cognitive mapping across domains, our understanding remains very limited. As noted by several scholars (e.g. Langfield-Smith & Wirth, 1992; Nadkarni & Narayanan, 2005; Wellman, 1994), complexity˜ and many additional features of cognitive map structure˜ remain unexploited and unexplained.

In future studies it will be illuminating to consider other – less common - measures indicative of cognitive map complexity, for example average chain length (Jenkins & Johnson, 1997a). A chain refers to a sequential set of perceived construct-to-construct links and calculating the mean length of all complete chains within a given map derives average chain length. For example, in the case of a map containing two chains, one comprising four links, the other two, the average chain length would be three. The greater the average chain length, the greater the elaboration in terms of explanations for the patterns of events or actions depicted in the map.

Moreover, other structural properties of cognitive maps also remain little understood, for example ‘centrality’ - whereby a highly centralised map is focused on a single construct with most constructs in the map being directly or indirectly connected to this central construct. Nadkarni and Naryanan (2005) suggest that centrality may be closely related to components of general cognitive ability such a logical reasoning rather than domain understanding, and it will be interesting to determine whether this is a valid line of reasoning.

As recently concluded by Narayanan and Liao (2005: 369), the analysis of the behaviour – the predictions or analysis of decisions or actions that one can make on the basis of cognitive maps – remains the “Holy Grail”. Our chosen complexity measure and our simulation method were chosen to provide maximum utility for the collective and cumulative process of theory building and theory testing for researchers working across domains.

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